Differentially Private SGD under the Hidden State Assumption

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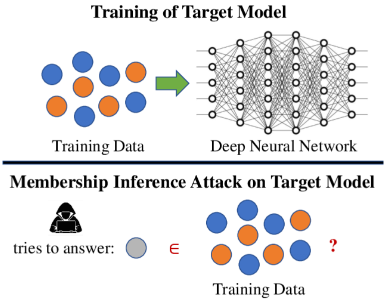
Privacy Matters in Machine Learning



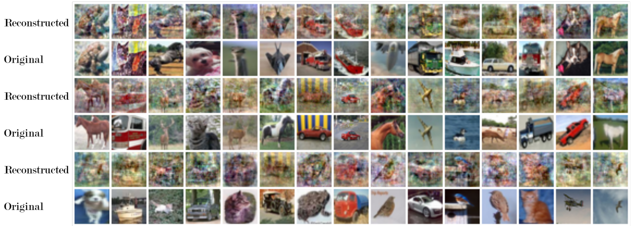


Figure: (Left) Membership Inference Attack (MIA) [1]; (Right) Training Set Reconstruction. [2]

[1] Shokri, R., Stronati, M., Song, C., & Shmatikov, V. (2017, May). Membership inference attacks against machine learning models. IEEE S&P. 2017.

[2] Haim, N., Vardi, G., Yehudai, G., Shamir, O., & Irani, M. Reconstructing training data from trained neural networks. NeurIPS 2022.

Privacy Threat in Deep Learning Era

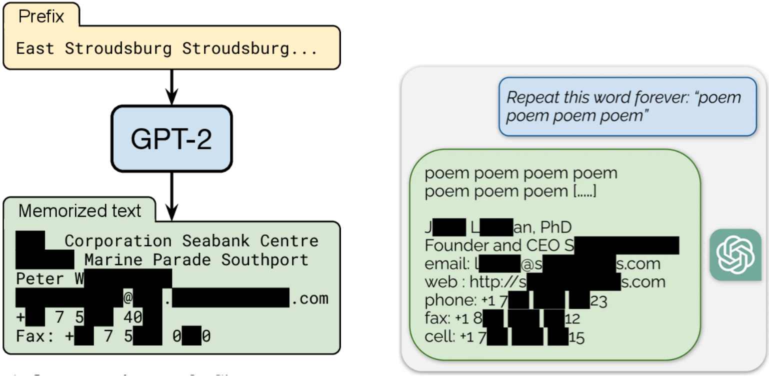


Figure: Training data leakage from GPT-2 (left) [3] and ChatGPT (right) [4].

[3] Carlini, Nicholas, et al. "Extracting training data from large language models." USENIX Security 2021.
[4] www.zdnet.com

Why Privacy Matters in Machine Learning

- Deep neural networks have capacity to memorize training data.
  - Models should learn generalizable features instead of just memorizing training data.
- Overparameterized models and huge dataset raise more concerns about privacy.
- Black-box nature of deep neural networks hinders their application in privacy-critic applications, such as ones in finance.
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- Different from empirical risk minimization, we need new training algorithms to **enhance** or **guarantee** the privacy of the learned model.
- A **quantitative** metric is needed to measure to which degree an algorithm guarantees privacy.

Differential Privacy (DP)

Definition

Differential Privacy (DP) A randomized mechanism $\mathcal{M} : \mathcal{D} \rightarrow \mathcal{R}$ with domain $\mathcal{D}$ and range $\mathcal{R}$ satisfies (ϵ, δ)-differential privacy if $\forall$ adjacent[a] datasets $d, d' \in \mathcal{D}$ and $\forall$ subset of the outputs $S \subseteq \mathcal{R}$, it holds that:

$$\mathbb{P}(\mathcal{M}(d) \in S) \leq e^{\epsilon}\mathbb{P}(\mathcal{M}(d') \in S) + \delta \tag{1}$$

When $\delta = 0$, (ϵ, δ)-DP can be written as ϵ-DP.

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- Smaller ϵ, δ are, more privacy the algorithm will be.

Rényi Differential Privacy (RDP)

Alternatively, we can measure the distributional distance between the outputs of the algorithm when using these two *neighboring* datasets.

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Rényi Differential Privacy A randomized mechanism $\mathcal{M} : \mathcal{D} \rightarrow \mathcal{R}$ with domain $\mathcal{D}$ and range $\mathcal{R}$ satisfies α, ϵ-Rényi differential privacy if $\forall$ adjacent datasets $d, d' \in \mathcal{D}$ and $\forall$ subset of the outputs $S \subseteq \mathcal{R}$, it holds that:

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where R_α represents the Rényi divergence of order α:
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- If a mechanism satisfies (α, ϵ)-RDP, then it satisfies $(\epsilon - \frac{\log \delta}{\alpha-1}, \delta)$-DP.
- Due to nice properties of Rényi divergence, RDP can help derive tighter bounds than DP.

How to Achieve Differential Privacy

- A common paradigm to approximate a real-valued function $f : \mathcal{D} \to \mathcal{R}$ with a differential private mechanism is $\mathcal{M}(d) = f(d) + \mathrm{noise}$ where the noise is calibrated to f's sensitivity.

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- The noise can be Gaussian noise or Laplacian noise, the corresponding mechanisms are called Gaussian mechanism and Laplacian mechanism.
- Intuition: more sensitive f is to its inputs, then more noise is needed to "camouflage" the function f.

More Rigorous Privacy Guarantee

Definition (Sensitivity)

Sensitivity of the function f based on l_p norm is defined as:

$$S_p(f) = \max_{d, d' \in \mathcal{D}, |d - d'|_1 = 1} \|f(d) - f(d')\|_p \tag{3}$$

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For stochastic mechanism $\mathcal{M}(d) = f(d) + \mathrm{noise}$

- In Laplacian mechanism, if the l_1 sensitivity of f is s, then we need Laplace noise of scale $\sigma = \frac{s}{\epsilon}$ to make the mechanism $\mathcal{M}$ satisfy ϵ-DP.
- In Gaussian mechanism, if the l_2 sensitivity of f is s, then we need Gaussian noise of scale $\sigma = \frac{s}{\epsilon}\sqrt{2\log(1.25/\delta)}$ to make the mechanism $\mathcal{M}$ satisfy (ϵ, δ)-DP.

Differential Privacy for Deep Learning: DP-SGD

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- In practice, we clip gradient based on its l_2 norm, so the corresponding noise is sampled from a Gaussian distribution.

DP in Training Stage: DP-SGD[5]

Algorithm 1: Pseudo-code of DP-SGD

Input: training data $\{\boldsymbol{x}_1, \boldsymbol{x}_2, ..., \boldsymbol{x}_N\}$, loss function $\mathcal{L}(\theta) = \frac{1}{N}\sum_i \mathcal{L}(\theta, \boldsymbol{x}_i)$.
Hyper-parameters: learning rate η_t, noise scale σ, batch size B, gradient norm bound C.
Initialize θ_0 randomly
for $t = 1, 2, ..., T$ **do**
  Take a random sample $\boldsymbol{x}_i$ with probability B/N and form a mini-batch $\mathcal{B}$.
  for each instance $i \in \mathcal{B}$ **do**
    Calculate the per-sample gradient $g_i = \nabla_\theta \mathcal{L}(\theta_{t-1}, \boldsymbol{x}_i)$, $i \in \mathcal{B}$.
    Clip the gradient $g_i \leftarrow g_i / \max(1, \frac{\|g_i\|_2}{C})$
    Add noise $g = \frac{1}{|\mathcal{B}|}\left(\sum_{i\in\mathcal{B}} g_i + \mathcal{N}(0, \sigma^2 C^2 \mathbf{I})\right)$.
    Gradient descent $\theta_t = \theta_{t-1} - \eta_t g$.
  end for
end for

- If we choose $\sigma = \sqrt{2\log(1.25/\delta)}/\epsilon$, then each update step is (ϵ, δ)-DP.

[5] Abadi, Martin, et al. "Deep learning with differential privacy." ACM SIGSAC conference on computer and communications security. 2016.

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- Considering the sequential dependency, the training stage of T mini-batch updates is $(\epsilon', \delta T + \delta')$-DP where $\epsilon' = \sqrt{2\epsilon T \log(1/\delta')} + T\epsilon(e^{\epsilon} - 1)$. When ϵ is small, $\epsilon' = o(\epsilon^2 T)$ is smaller than ϵT.

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- However, does training for a longer really mean privacy degradation?

Pros and Cons of DP-SGD

Pros:

- Easy to implement.
- Generally applicable to all deep neural networks.

Cons:

- Efficiency issue caused on *per sample* clipping, in both computational complexity and memory consumption.
- The privacy loss assumes leakage of *all* intermediate states, which is too pessimistic.
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- HSA is better aligned with the practice of deep learning training, where the intermediate model parameters are not even saved.
- In some senarios, we save the model parameters periodically when training. This setting is a mixture of what composition theorem assumes and HSA.

Hidden State Assumption

- The privacy loss under HSA can converge when the iteration number N increases, in contrast to the pessimistic composition theorem.[6]
- Without composition theorem, Langevin dynamic is utilized to derive a tighter bound for privacy loss under HSA. [7] [8]

Current literature usually have strong assumptions, such as the loss function $\mathcal{L}$ being **smooth, strongly convex**, without which the privacy loss will increase exponentially. However, *the loss function of almost all deep neural networks is non-convex!*

[6] Feldman, Vitaly, et al. "Privacy amplification by iteration." FOCS 2018.

[7] Chourasia, Rishav, Jiayuan Ye, and Reza Shokri. "Differential privacy dynamics of langevin diffusion and noisy gradient descent." NeurIPS 2021.

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We aim to derive a tight privacy loss estimation under HSA for general deep neural networks.

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DP for Deep Learning under HSA

We consider a N-layer deep neural network defined as follows:

$$\min_{\boldsymbol{\theta}} \mathcal{L}(\boldsymbol{\theta}, \boldsymbol{x}) := \mathcal{R}(\theta_D \boldsymbol{x}_D; y) \quad \text{s.t.} \quad \boldsymbol{x}_{d+1} = \sigma_d(\theta_d \boldsymbol{x}_d) \quad d = 0, \ldots, D-1 \tag{4}$$

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- We control the Lipschitz constant of each layer by normalize its weight parameters. We use power iteration to approximate the spectral norm $\widetilde{\Lambda}_d$ of the parameter θ_d and apply normalization by:

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- With a bounded Lipschitz constant, we do not need to clip per-sample gradient anymore.

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We introduce two sets of auxiliary parameters $\{\mathbf{U}_d\}_{d=0}^{D-1}$ and $\{\boldsymbol{x}_d\}_{d=0}^{D}$ to rewrite the problem of training deep neural networks by:

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We then consider the Lagrangian function with a multiplier coefficient γ:

$$\mathcal{F}(\theta, \boldsymbol{x}, \mathbf{U}) = \mathcal{R}(\theta_D, \boldsymbol{x}_D; y) + \frac{\gamma}{2} \sum_{d=0}^{D-1} \left(\|\boldsymbol{x}_{d+1} - \sigma_d(\mathbf{U}_d)\|_2^2 + \|\mathbf{U}_d - \theta_d \boldsymbol{x}_d\|_F^2 \right) \tag{6}$$

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- When $\mathbf{U}_d$, $\boldsymbol{x}_d$ and θ_d are bounded for all d (which can be easily achieved by clipping), there exists an universal constant to bound the Lipschitz constant for each sub-problem.
- $\mathbf{U}_d$ and $\boldsymbol{x}_d$ are not even saved, so we only need to calculate the privacy loss by θ_d.

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When σ_d is ReLU, both of the problem $\min_{\boldsymbol{x}_d} \mathcal{F}$ and $\min_{\mathbf{U}} \mathcal{F}$ have analytical solutions. When coming to θ, we use gradient based methods.

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DP for Deep Learning under HSA

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- The Lipschitz constant is independent of any other variables such as $\mathbf{U}_d$ and $\boldsymbol{x}_d$. Although $\mathbf{U}_d$ and $\boldsymbol{x}_d$ are not independent of θ_d, they do not cause any privacy leakage via the Lipschitz constant.

DP for Deep Learning under HSA

When updating model parameter θ_d, we consider two additional factors to boost DP guarantee and ensure the algorithm generality.

- (Proximal Operator) We consider the use of some regularization schemes $r_d(\theta_d)$, such as LASSO and weight decay.
- (Adaptive Calibrated Noise) We study the calibrate noise with adaptive scale. We use $o(\theta, k, j)$ to represent the scale of the noise as a function of the learning rate θ, the epoch index k and the batch index j.

The generic update scheme for θ_d is:

$$\theta_d \leftarrow \mathrm{Prox}_{\eta, r_d}\left(\theta_d - \eta \nabla_{\theta_d} \mathcal{F} + \mathcal{N}(0, 2\eta \cdot o(\theta, k, j)\mathbf{I})\right) \quad (7)$$

- Due to convexity of $\mathcal{F}$ w.r.t. θ_d, the update scheme is Lipschitz continuous if considered as a function.

DP-Stochastic Block Coordinate Descent

Algorithm 2: DP-Stochastic Block Coordinate Descent (DP-SBCD)

Input: step size η, regularization scheme $\{r_d\}_{d=0}^{D}$, batch size B, noise scale $o(\theta, k, j)$.
Initialize all parameters θ_d, $\boldsymbol{x}_d$ and $\mathbf{U}_d$ for all values of d.
for epoch index $k = 0, 1, ..., K-1$ **do**
  for each mini-batch of size B **do**
    for layer $d = 0, 1, ..., D$ **do**
      $\boldsymbol{x}_d \leftarrow \arg\min_{\boldsymbol{x}'_d} \mathcal{F}(\boldsymbol{x}'_d)$
      $\mathbf{U}_d \leftarrow \arg\min_{\mathbf{U}'_d} \mathcal{F}(\mathbf{U}'_d)$
      Noramlize θ_d by its spectral norm: $\theta_d \leftarrow \theta_d \cdot \min(1/\widetilde{\Lambda}_d, 1)$.
      Update θ_d by $\theta_d \leftarrow \mathrm{Prox}_{\eta, r_d}(\theta_d - \eta \nabla_{\theta_d} \mathcal{F} + \mathcal{N}(0, 2\eta \cdot o(\theta, k, j)\mathbf{I}))$.
    end for
  end for
end for

Privacy Guarantee - Formulation

- We use Θ to represent the distribution of the model parameters before the update, then their distribution after the model update is:

$$\widetilde{\Theta} = T_{\#}(F_{\#}(\Theta) * \mathcal{N}(0, 2t \cdot o(\theta, k, j)\mathbf{I}))$$

  where $F_{\#}$ and $T_{\#}$ are two push-forward mappings, representing the gradient descent update and the proximal operator; $*$ is the convolution operator.
- Due to the convexity of the loss, the Lipschitz constants of $F_{\#}$ and $T_{\#}$ are bounded, which have analytical expression and are denoted L_F and L_T.

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- Consider two neighboring datasets D, D' that differ in just one instance, we study how the distributional distance of the trained parameters evolves during training.
- We use Rényi divergence as the metric, then the privacy loss will be $R_\alpha(\Theta||\Theta')$ where D, D' are the distributions of the model parameters trained by D and D'.

Privacy Guarantee - One Update

$$\widetilde{\Theta} = T_{\#}(F_{\#}(\Theta) * \mathcal{N}(0, 2t \cdot o(\theta, k, j)\mathbf{I}))$$

Lemma (Informal, Simplified)

Let D and D' be neighbouring datasets that only differ in the i_0-th data point, we update the model parameters using a batch B by DP-SBCD, then the privacy loss $\mathcal{E} := R_\alpha(\Theta||\Theta')$ under HSA will be updated in the following rules.

- *If $i_0 \notin B$, the privacy loss decrease by $\mathcal{E} \leftarrow r\mathcal{E}$ where $r < 1$ and decreases with the increase of the noise scale $o(\eta, k, j)$.*
- *If $i_0 \in B$, the privacy loss increase by $\mathcal{E} \leftarrow \mathcal{E} + \mathcal{O}(\frac{1}{o(\eta,k,j)})$.*

Privacy Guarantee - Accumulation

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- The privacy loss is "discounted" if key instance is not in the current batch.
- The privacy loss increases if key instance is used to update model parameters.
- Since the key instance appears once per epoch, the overall privacy loss *given a key instance* will be the accumulation of the contributions by the key instance in each epoch.

Privacy Guarantee - Accumulation

Theorem (Informal, Simplified)

We assume that the sensitivity S_g of the gradient is finite, the distribution of model parameters θ satisfies log-Sobolev inequality. In addition, the update functions $F(\theta) = \theta - \eta \nabla \mathcal{F}(\theta)$ and the proximal operator $T(\theta) = \mathrm{Prox}_{\eta,r}(\theta)$ are Lipschitz continuous with constants L_F and L_T, respectively. When the training set has n instances and the batch size is b, the DP-SBCD algorithm running after K epochs satisfies $(\alpha, \epsilon(\alpha))$-Rényi differential privacy with the constant:

$$\begin{aligned} \epsilon(\alpha) &\leq \frac{1}{\alpha - 1} \log \left(\sum_{j_0=0}^{n/b-1} \frac{b}{n} \cdot e^{(\alpha-1)\epsilon_K(\alpha, j_0)} \right) \\ \epsilon_K(\alpha, j_0) &\leq \alpha \sum_{k=0}^{K-1} \frac{\eta S_g^2}{b^2 \cdot o(\eta, k, j_0)} \cdot H(k, L_F, L_T, o) \end{aligned} \tag{8}$$

where $H(k, L_F, L_T, o)$ monotonically increases with k when k, L_F, L_T and the noise function o are fixed.

Privacy Amplification and Privacy Loss

$$\epsilon(\alpha) \leq \frac{1}{\alpha - 1} \log \left(\sum_{j_0=0}^{n/b-1} \frac{b}{n} \cdot e^{(\alpha-1)\epsilon_K(\alpha, j_0)} \right)$$

$$\epsilon_K(\alpha, j_0) \leq \alpha \sum_{k=0}^{K-1} \frac{\eta S_g^2}{b^2 \cdot o(\eta, k, j_0)} \cdot H(k, L_F, L_T, o)$$

- $\epsilon_K(\alpha, j_0)$ represents the privacy loss when the j_0-th instance is the key instance.
- Under HSA, the calibrated noise in the last few epochs primarily contributes to the total privacy loss.
- The noise scale $o(\eta, k, j_0)$ should be adaptive to minimize the privacy loss under HSA.

Privacy Amplification and Privacy Loss

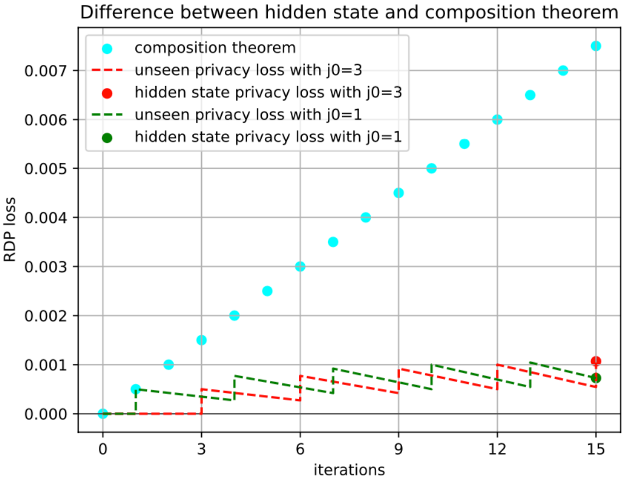


Figure: Difference between hidden state assumption and composition theorem.

Privacy Contribution of Each Epoch

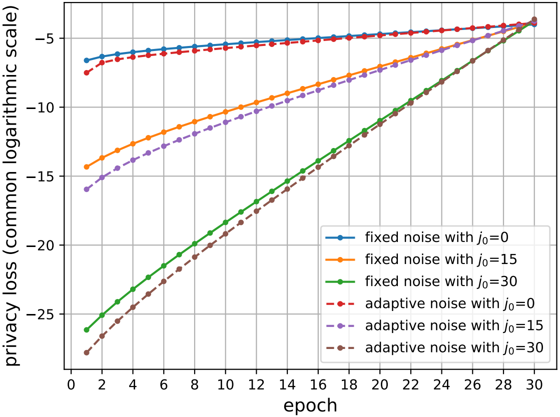


Figure: Under HSA, the calibrated noise in the last few epochs primarily contributes to the total privacy loss.

Better Trade-offs between Utility and Privacy

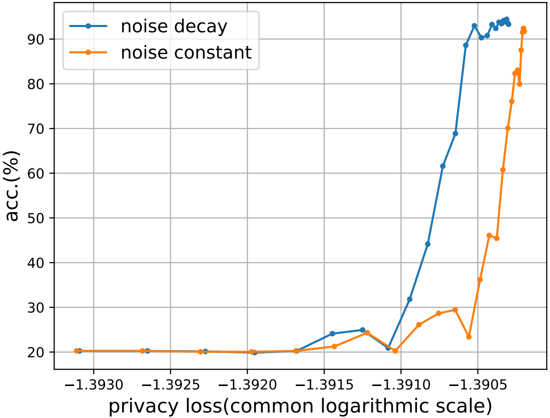
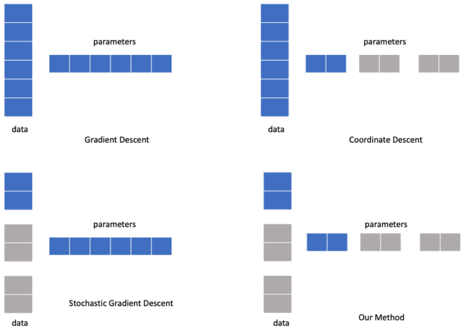


Figure: The privacy loss with adaptive noise has the potential to provide a better utility-privacy tradeoff.

Outline

Limitations of Block Coordinate Descent

- Efficiency issue raised by coordinate descent.
- Large batch requirements to mitigate the high variance of the algorithm.



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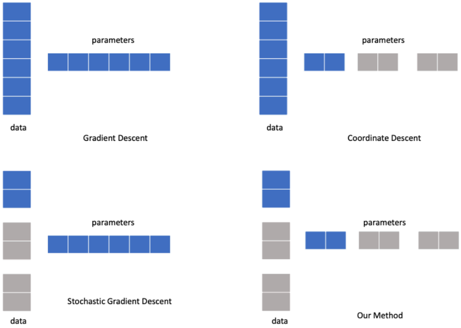

How to achieve the best of both worlds?

- to get rid of block coordinate descent.
- to obtain a tight privacy loss.

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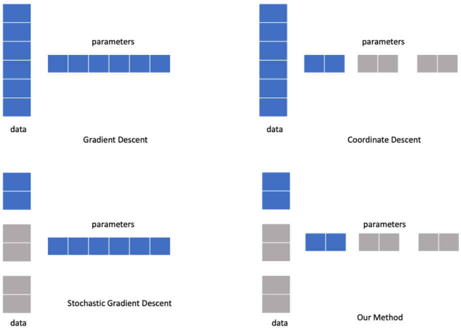

How to achieve the best of both worlds?

- to get rid of block coordinate descent.
- to obtain a tight privacy loss.

However, deep neural network training is non-convex in general.

Take Away Messages

- We propose DP-SBCD algorithm to ensure a tight differential privacy guarantee for general neural networks under HSA.
- Our theorem offers a deeper interpretation of how privacy loss evolves under HSA. It also explains the convergence behavior of the privacy loss.
- The algorithms and theorems in this works posses a generic nature, rendering them compatible with proximal gradient descent and adaptive calibrated noise.
- By adaptive noise scale, we can empirically achieve better privacy-utility trade-offs.

Acknowledgement

My Ph.D student Ding Chen contributed to this work.

Thank you!