

Finding the Mixed Nash Equilibria of Generative Adversarial Networks

Ya-Ping Hsieh
EPFL

Chen Liu
EPFL

Volkan Cevher
EPFL

① Motivation

Problem formulation for training Wasserstein GANs (WGAN):

$$\min_{\theta \in \Theta} \max_{\mathbf{w} \in \mathcal{W}} \mathbb{E}_{X \sim \mathbb{P}_{real}} [f_{\mathbf{w}}(X)] - \mathbb{E}_{X \in \mathbb{P}_{\theta}} [f_{\mathbf{w}}(X)]$$

But, training GANs is not easy.

- Highly unstable training.
- Lack of theory about the existence of saddle points.

② Proposal: Mixed Nash Equilibria (NE)

WGAN with mixed strategy ^a

$$\min_{\nu \in \mathcal{M}(\Theta)} \max_{\mu \in \mathcal{M}(\mathcal{W})} \mathbb{E}_{\mathbf{w} \sim \nu} \mathbb{E}_{X \sim \mathbb{P}_{real}} [f_{\mathbf{w}}(X)] - \mathbb{E}_{\mathbf{w} \sim \mu} \mathbb{E}_{\theta \sim \nu} \mathbb{E}_{X \sim \mathbb{P}_{\theta}} [f_{\mathbf{w}}(X)]$$

Define $g(\mathbf{w}) = \mathbb{E}_{X \sim \mathbb{P}_{real}} [f_{\mathbf{w}}(X)]$, $G\nu(\mathbf{w}) = \mathbb{E}_{\theta \sim \nu, X \sim \mathbb{P}_{\theta}} [f_{\mathbf{w}}(X)]$ and $\langle \nu, h \rangle = \mathbb{E}_{\nu} h$, WGAN with mixed strategy can be reformulated

$$\min_{\nu \in \mathcal{M}(\Theta)} \max_{\mu \in \mathcal{M}(\mathcal{W})} \langle \mu, g \rangle - \langle \mu, G\nu \rangle$$

This is exactly an *infinite* dimensional two-player game.

How to find Mixed NE? $\triangleright$ **Entropic Mirror Descent (MD)**

Start with a two-player game with *finite* actions

$$\min_{\mathbf{p} \in \Delta_m} \max_{\mathbf{q} \in \Delta_n} \langle \mathbf{q}, \mathbf{a} \rangle - \langle \mathbf{q}, \mathbf{A}\mathbf{p} \rangle$$

Entropic MD learns an $O(T^{-1/2})$ -NE in T iterations.

$$\begin{cases} \mathbf{p}_{t+1} = MD_{\eta}(\mathbf{p}_t, -\mathbf{A}^T \mathbf{q}_t) \\ \mathbf{q}_{t+1} = MD_{\eta}(\mathbf{q}_t, -\mathbf{a} + \mathbf{A}\mathbf{p}_t) \end{cases}$$

Here, MD_{η} is the MD iterate defined by entropy function $\phi(\mathbf{z}) = \sum_{i=1}^d z_i \log z_i$ and its Fenchel dual $\phi^*(\mathbf{y}) = \log \sum_{i=1}^d e^{y_i}$.

$$\mathbf{z}' \equiv MD_{\eta}(\mathbf{z}, \mathbf{b}) = \nabla \phi^*(\nabla \phi(\mathbf{z}) - \eta \mathbf{b})$$

$$\mathbf{z}'_i = \frac{\mathbf{z}_i e^{-\eta \mathbf{b}_i}}{\sum_{i=1}^d \mathbf{z}_i e^{-\eta \mathbf{b}_i}}$$

Can **Entropic MD** generalize to *infinite* dimensional two-player game formulation of WGAN and enjoy the same convergence rate?

Answer is Yes!

^aAlthough we use WGAN here as example, our methods can be applied to other GANs as well.

③ Main Theory

ENTROPIC MD FOR MIX-STRATEGY WGAN

Initial distribution μ_1, ν_1 , learning rate η given

for $t = 1, 2, \dots, T$ **do**

$$\nu_{t+1} = MD_{\eta}(\nu_t, -G^T \mu_t) \quad \triangleright \text{Update } \nu$$

$$\mu_{t+1} = MD_{\eta}(\mu_t, -g + G\nu_t) \quad \triangleright \text{Update } \mu$$

return $\bar{\mu} = \frac{1}{T} \sum_{t=1}^T \mu_t$ and $\bar{\nu} = \frac{1}{T} \sum_{t=1}^T \nu_t$.

Main Theorem (informal)

Assume $-G^T \mu_t$ and $-g + G\nu_t$ are bounded and smooth.

1. If we have access to deterministic values of $-G^T \mu_t$ and $-g + G\nu_t$, then Entropic MD achieves $O(T^{-1/2})$ - NE in T iterations with proper learning rate.
2. If we have access to unbiased stochastic values with bounded variance of $-G^T \mu_t$ and $-g + G\nu_t$, then Entropic MD achieves $O(T^{-1/2})$ - NE in expectation in T iterations with proper learning rate.

⑤ Experiments

We name our method Mirror-GAN and compare it with other method to train GANs.

Iterations

25 Mixed Gaussians

Iterations

LSUN Bedroom

SGD

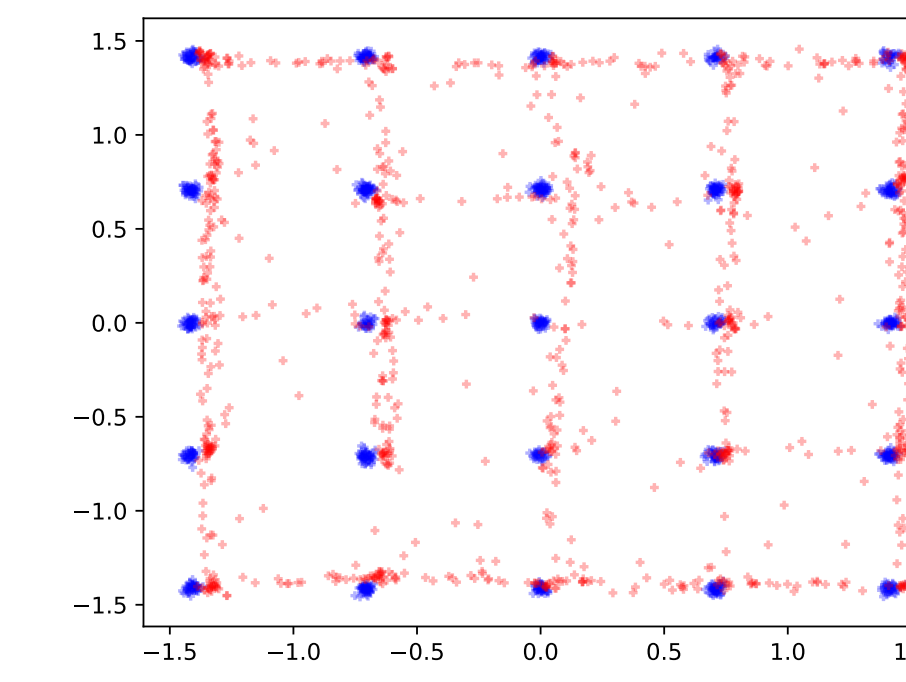
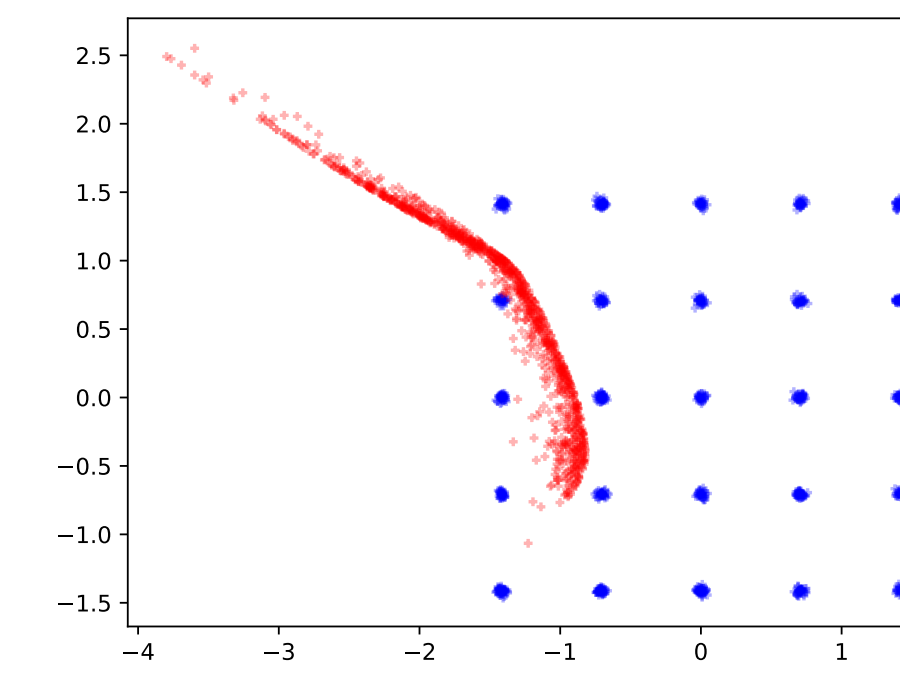
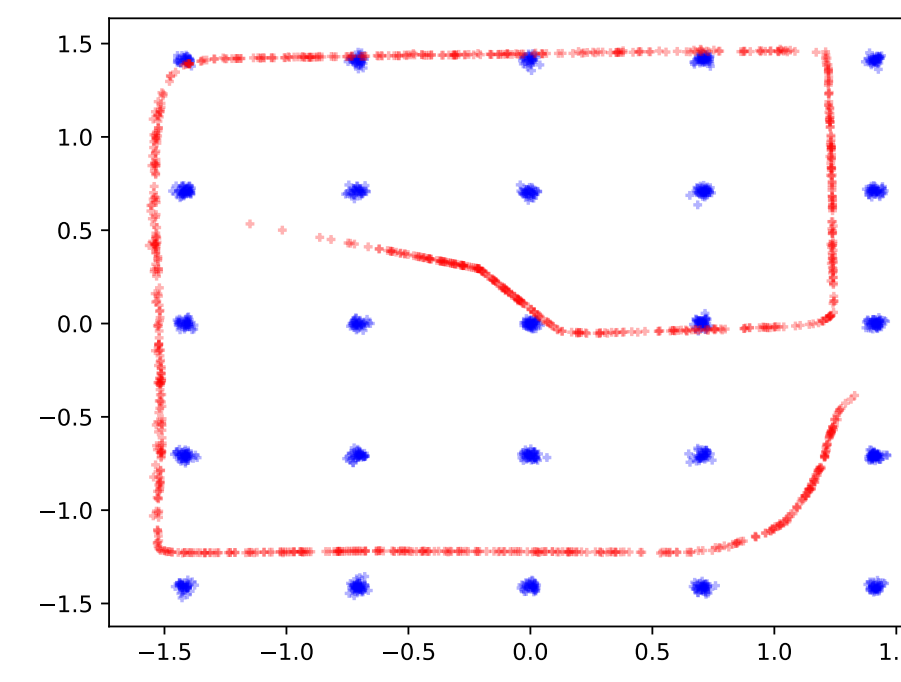
Adam

Mirror Descent

RMSProp

Mirror Descent

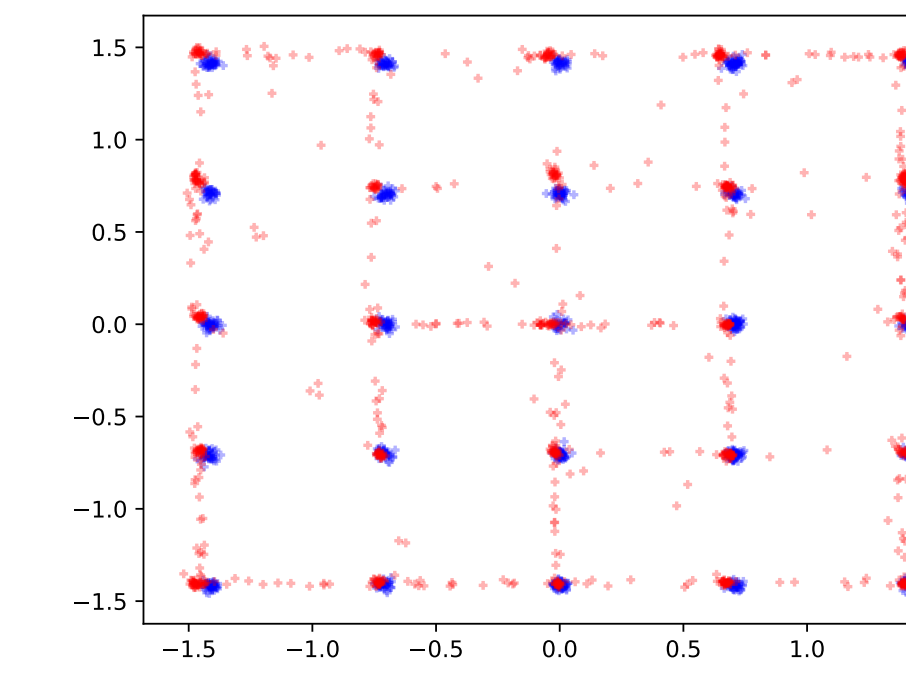
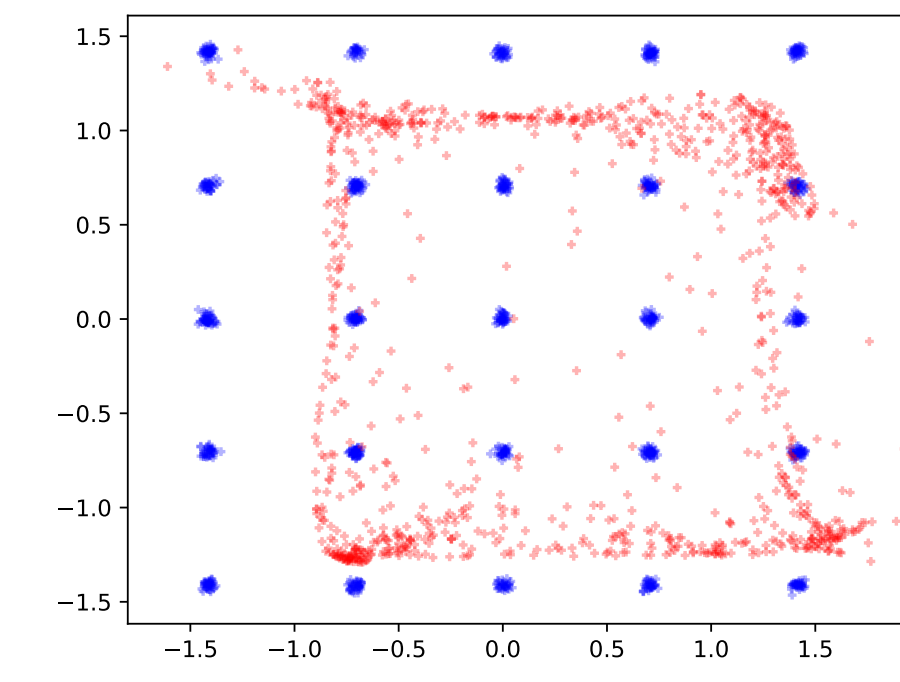
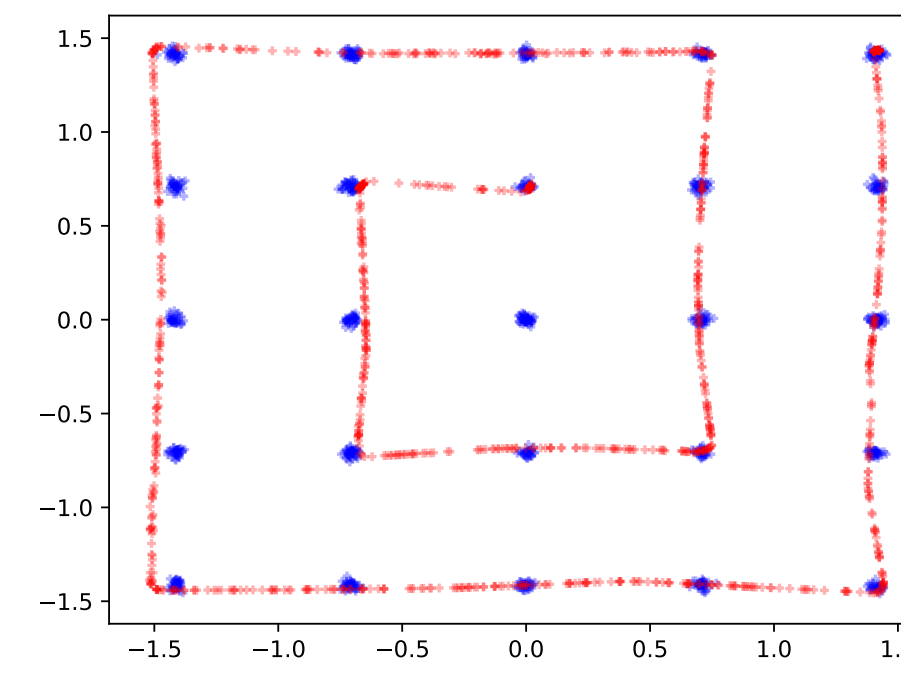
2×10^4



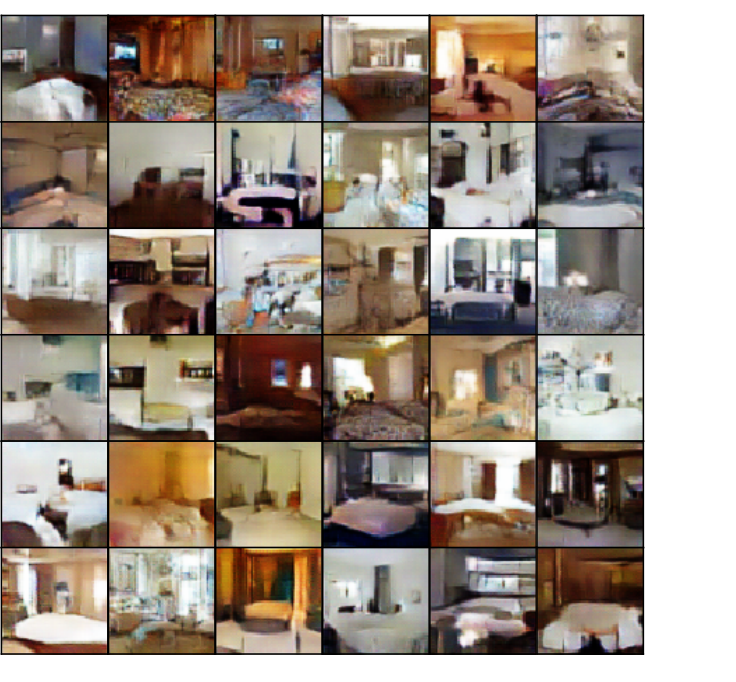
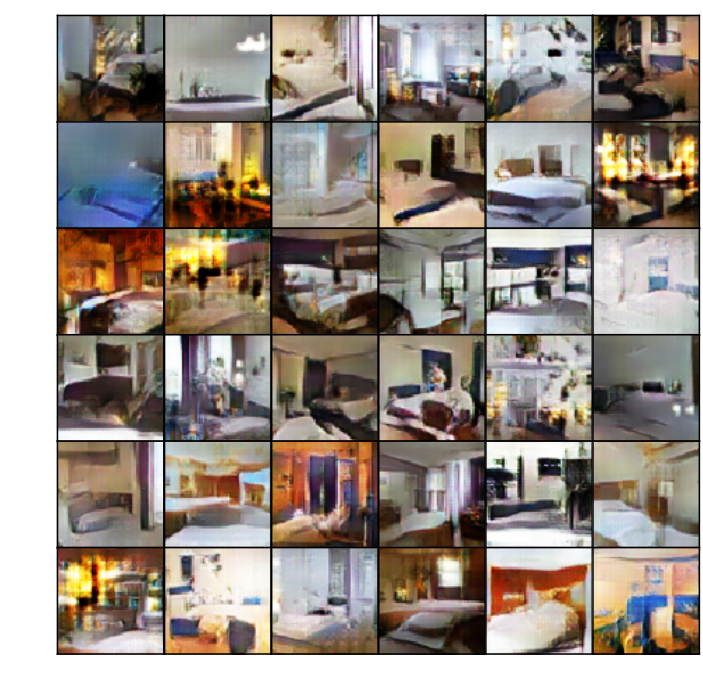
4×10^4



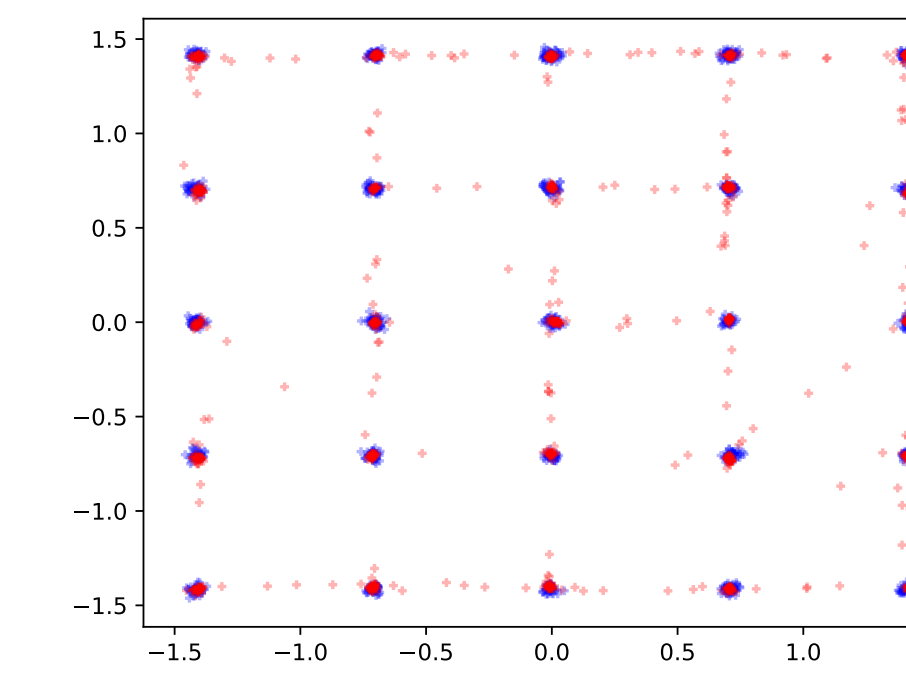
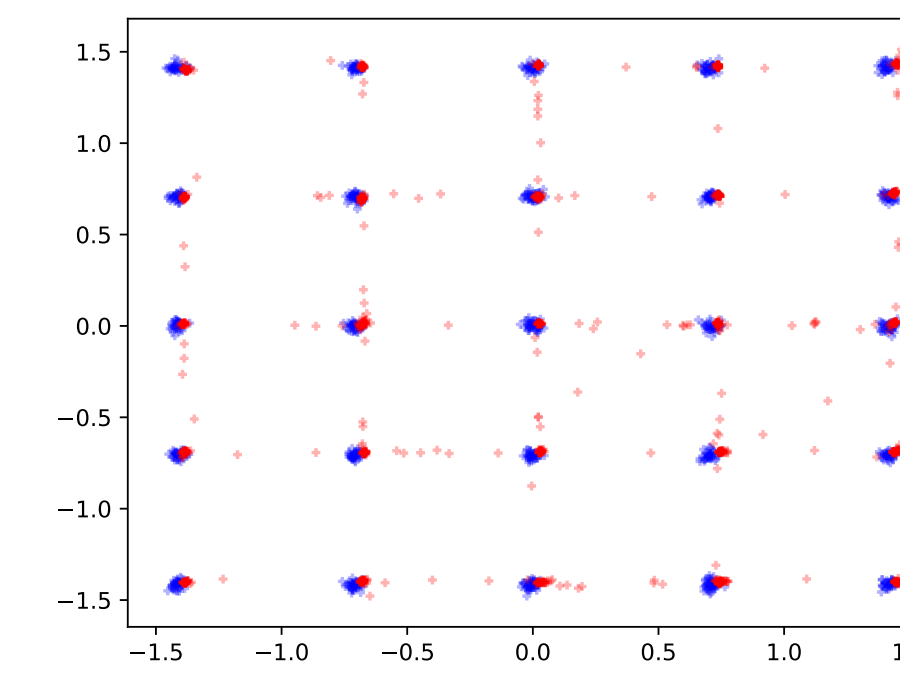
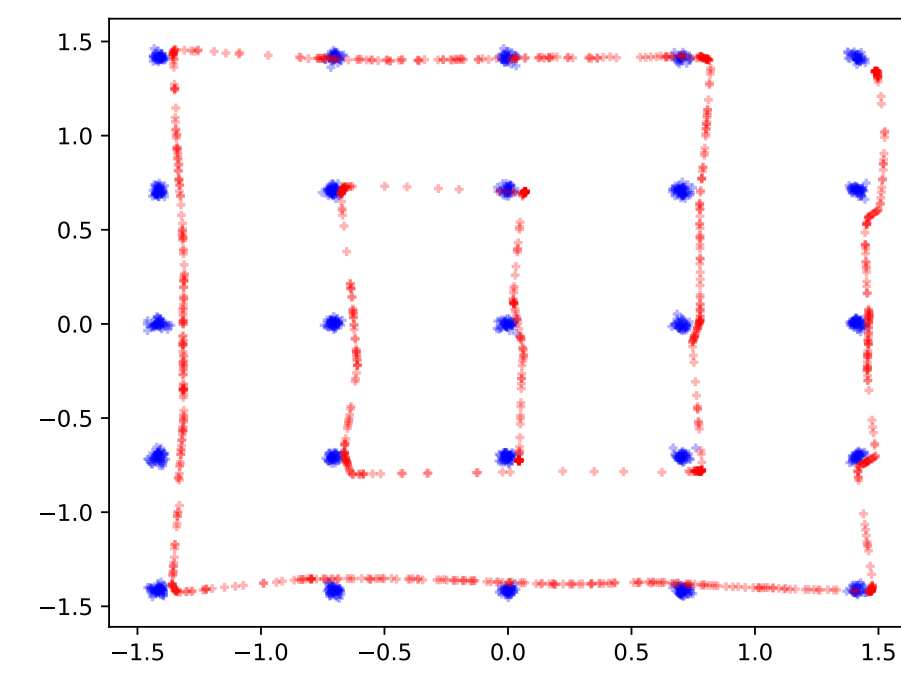
5×10^4



8×10^4



10×10^4



10×10^4

