DualOptim+: Bridging Shared and Decoupled Optimizer States for Better Machine Unlearning in Large Language Models

Xuyang Zhong [1] Qizhang Li [2] Yiwen Guo [2] Chen Liu [1]

[1]City University of Hong Kong [2]Independent Researcher

xuyang.zhong@my.cityu.edu.hk {liqizhang95, guoyiwen89}@gmail.com chen.liu@cityu.edu.hk

Comparison of Baselines and DualOptim+

Machine unlearning (MU) solves the optimization problem:

$$\min_{\boldsymbol{\theta}} \mathcal{L}_f(\boldsymbol{\theta}, \mathcal{D}_f) + \mathcal{L}_r(\boldsymbol{\theta}, \mathcal{D}_r) \tag{1}$$

The update rules of baselines and DualOptim+ are presented as follows:

- **Joint:**

$$\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} - \mathcal{P}(\nabla_{\boldsymbol{\theta}}(\mathcal{L}_f + \mathcal{L}_r)). \tag{2}$$

- **Alternate:**

$$\begin{cases} \boldsymbol{\theta} \leftarrow \boldsymbol{\theta} - \mathcal{P}(\nabla_{\boldsymbol{\theta}}\mathcal{L}_f) & \text{for forget data} \\ \boldsymbol{\theta} \leftarrow \boldsymbol{\theta} - \mathcal{P}(\nabla_{\boldsymbol{\theta}}\mathcal{L}_r) & \text{for retain data} \end{cases}. \tag{3}$$

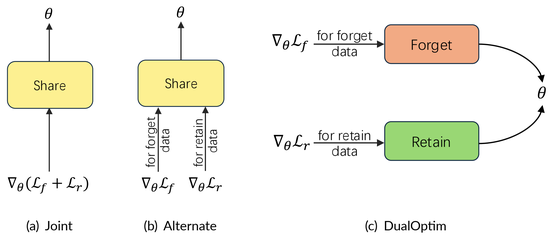
- **DualOptim:**

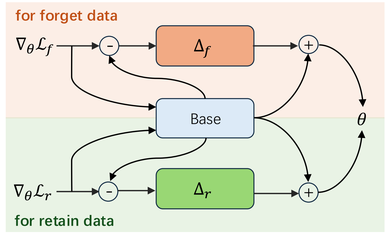
$$\begin{cases} \boldsymbol{\theta} \leftarrow \boldsymbol{\theta} - \mathcal{P}_f(\nabla_{\boldsymbol{\theta}}\mathcal{L}_f) & \text{for forget data} \\ \boldsymbol{\theta} \leftarrow \boldsymbol{\theta} - \mathcal{P}_r(\nabla_{\boldsymbol{\theta}}\mathcal{L}_r) & \text{for retain data} \end{cases}. \tag{4}$$

- **DualOptim+ (Ours):**

$$\begin{cases} B \leftarrow \beta B + (1-\beta)\nabla_{\boldsymbol{\theta}}\mathcal{L}_f & \text{for forget data} \\ B \leftarrow \beta B + (1-\beta)\nabla_{\boldsymbol{\theta}}\mathcal{L}_r & \text{for retain data} \end{cases}. \tag{5}$$

$$\begin{cases} \Delta_f \leftarrow \beta\Delta_f + (1-\beta)(\nabla_{\boldsymbol{\theta}}\mathcal{L}_f - \widehat{B}) & \text{for forget data} \\ \Delta_r \leftarrow \beta\Delta_r + (1-\beta)(\nabla_{\boldsymbol{\theta}}\mathcal{L}_r - \widehat{B}) & \text{for retain data} \end{cases}. \tag{6}$$





(d) DualOptim+

Figure 1. Comparison of baselines and DualOptim+.

Pseudo Code

The pseudo code of **DualOptim+** with Adam is shown below.

Algorithm 1 DualOptim+ with AdamW

Input: parameter $\boldsymbol{\theta}$, learning rate η, betas (β_1, β_2), epsilon ϵ, weight decay factor λ, forget objective $\mathcal{L}_f$, retain objective $\mathcal{L}_r$, total steps N, forget frequency F_f, retain frequency F_r
Initialize: $\boldsymbol{m}_{\Delta_f} \leftarrow 0$, $\boldsymbol{m}_{\Delta_r} \leftarrow 0$, $\boldsymbol{m}_B \leftarrow 0$, $\boldsymbol{v}_{\Delta_f} \leftarrow 0$, $\boldsymbol{v}_{\Delta_r} \leftarrow 0$, $\boldsymbol{v}_B \leftarrow 0$, $t_f \leftarrow 0$, $t_r \leftarrow 0$
$\widehat{\boldsymbol{m}}_B,\ \widehat{\boldsymbol{v}}_B \leftarrow \boldsymbol{m}_B,\ \boldsymbol{v}_B$
for $t = 1$ **to** N **do**
  if $t \bmod (F_f + F_r) \leq F_f$ **then**
    $t_f \leftarrow t_f + 1$
    $\boldsymbol{g},\ \boldsymbol{m}_\Delta,\ \boldsymbol{v}_\Delta,\ t' \leftarrow \nabla_{\boldsymbol{\theta}}\mathcal{L}_f(\boldsymbol{\theta}),\ \boldsymbol{m}_{\Delta_f},\ \boldsymbol{v}_{\Delta_f},\ t_f$
  else
    $t_r \leftarrow t_r + 1$
    $\boldsymbol{g},\ \boldsymbol{m}_\Delta,\ \boldsymbol{v}_\Delta,\ t' \leftarrow \nabla_{\boldsymbol{\theta}}\mathcal{L}_r(\boldsymbol{\theta}),\ \boldsymbol{m}_{\Delta_r},\ \boldsymbol{v}_{\Delta_r},\ t_r$
  end if
  $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} - \eta\lambda\boldsymbol{\theta}$
  $\boldsymbol{m}_\Delta \leftarrow \beta_1\boldsymbol{m}_\Delta + (1-\beta_1)(\boldsymbol{g} - \widehat{\boldsymbol{m}}_B)$
  $\boldsymbol{v}_\Delta \leftarrow \beta_2\boldsymbol{v}_\Delta + (1-\beta_2)(\boldsymbol{g}^2 - \widehat{\boldsymbol{v}}_B)$
  $\widehat{\boldsymbol{m}}_\Delta,\ \widehat{\boldsymbol{v}}_\Delta \leftarrow \boldsymbol{m}_\Delta/(1-\beta_1^{t'}),\ \boldsymbol{v}_\Delta/(1-\beta_2^{t'})$
  $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} - \eta(\widehat{\boldsymbol{m}}_B + \widehat{\boldsymbol{m}}_\Delta)/(\sqrt{|\widehat{\boldsymbol{v}}_B + \widehat{\boldsymbol{v}}_\Delta|} + \epsilon)$
  $\boldsymbol{m}_B \leftarrow \beta_1\boldsymbol{m}_B + (1-\beta_1)\boldsymbol{g}$
  $\boldsymbol{v}_B \leftarrow \beta_2\boldsymbol{v}_B + (1-\beta_2)\boldsymbol{g}^2$
  $\widehat{\boldsymbol{m}}_B,\ \widehat{\boldsymbol{v}}_B \leftarrow \boldsymbol{m}_B/(1-\beta_1^t),\ \boldsymbol{v}_B/(1-\beta_2^t)$
end for
Output: parameter $\boldsymbol{\theta}$

Theoretical Analysis on DualOptim+

Theorem 1 (Convergence of Base and Delta States). Assume the expectations of gradients $g_{f,t}$ and $g_{r,t}$ over time $\mathbb{E}_t[g_{f,t}] = mG$, $\mathbb{E}_t[g_{r,t}] = nG$ exist, where $m, n \in [-1, 1]$, and G is a non-negative constant. We have:

$$\begin{aligned} \lim_{T\to\infty} B_{(F_f+F_r)T} &= \frac{\beta^{F_r}(1-\beta^{F_f})m + (1-\beta^{F_r})n}{1-\beta^{F_f+F_r}}G, \\ \lim_{T\to\infty} \Delta_{f,(F_f+F_r)T} &= \frac{F_f\beta^{F_f-1}(1-\beta)\left(1-\beta^{F_r}\right)}{\left(1-\beta^{F_f}\right)\left(1-\beta^{F_f+F_r}\right)}(m-n)G, \\ \lim_{T\to\infty} \Delta_{r,(F_f+F_r)T} &= \frac{F_r\beta^{F_r-1}(1-\beta)\left(1-\beta^{F_f}\right)}{\left(1-\beta^{F_r}\right)\left(1-\beta^{F_f+F_r}\right)}(n-m)G. \end{aligned} \tag{7}$$

1. **Positive correlation:** When $m = n$, B converges to mG, and Δ_f and Δ_r converge to 0. **DualOptim+ acts like Alternate**.
2. **Negative correlation:** When $m = -\frac{1-\beta^{F_r}}{\beta^{F_r}(1-\beta^{F_f})}n$, B converges to 0. **DualOptim+ acts like DualOptim**.

Experiments

Table 1. Performance on **Fictitious Unlearning** task. The model is TOFU-tuned Phi-1.5.

Loss	Method	Phi 1.5 forget 1% data UFE ↑	TFE ↑	MU ↑	OVR ↑	forget 5% data UFE ↑	TFE ↑	MU ↑	OVR ↑	forget 10% data UFE ↑	TFE ↑	MU ↑	OVR ↑
IDK+GD	Joint	**78.11**	45.45	18.61	40.19	**72.55**	58.32	36.26	50.85	**71.65**	64.39	33.92	50.97
	Alternate	73.35	62.49	48.14	58.03	67.73	64.30	47.81	56.91	65.82	64.46	49.54	57.34
	DO	74.75	63.51	46.46	57.80	68.49	64.34	49.50	57.96	65.87	**66.84**	50.25	58.30
	DO 8bit	75.58	61.23	46.73	57.57	68.34	64.33	48.41	57.37	65.81	65.60	50.38	58.05
	DO+	75.51	67.85	**47.69**	**59.69**	67.63	**67.60**	**51.52**	**59.57**	65.42	66.50	**51.32**	**58.64**
	DO+ 8bit	73.69	**68.36**	47.53	59.28	67.56	65.94	50.36	58.55	65.26	65.59	51.30	58.36

Table 2. Performance on **Real-world Unlearning** task. The model is Llama-3-8B-Instruct.

Loss	Method	Llama 3 Unlearning Task UFE ↑	TFE ↑	MU ↑	OVR ↑	Downstream Tasks ARC-c ↑	MMLU ↑	TruthfulQA ↑	TriviaQA ↑	GSM8K ↑	AVG ↑
	Initial	30.55	–	61.45	46.00	55.38	64.59	37.33	50.93	76.12	56.87
IDK+GD	Joint	85.54	**72.96**	27.38	53.32	46.79	62.85	33.41	7.58	74.32	44.99
	Alternate	85.49	69.95	29.19	53.45	49.77	63.31	35.62	12.71	74.15	47.11
	DO	85.25	69.60	28.06	52.73	48.47	63.20	35.29	10.33	72.35	45.93
	DO 8bit	85.28	69.60	27.68	52.56	48.75	63.08	35.05	11.56	72.35	46.16
	DO+	**85.72**	69.94	27.96	52.90	50.85	64.43	36.35	11.17	**76.02**	47.77
	DO+ 8bit	85.47	69.59	**33.36**	**55.45**	**52.56**	**64.51**	**36.80**	**17.86**	75.21	**49.39**

Table 3. Performance on **Safety Alignment** task. The model is Alpaca-tuned Llama 3-8B-Instruct.

Method	Alpaca-Llama 3 Safety I-Mali ↑	I-CoNa ↑	I-Cont ↑	Q-Harm ↑	AVG ↑	Utility ARC-c ↑	MMLU ↑	TruthfulQA ↑	TriviaQA ↑	GSM8K ↑	AVG ↑	OVR ↑	XSTest ↓
Initial	28.00	38.76	55.00	64.00	46.44	45.56	52.53	29.74	12.11	13.12	30.61	33.56	0.40
Joint	94.67	96.63	**97.50**	97.00	96.45	47.04	51.63	33.74	12.18	14.10	31.74	54.84	**28.00**
Alternate	**97.00**	97.38	**97.50**	**99.67**	**97.89**	46.81	50.83	**34.60**	13.83	12.94	31.80	55.67	29.20
DO	95.67	97.94	**97.50**	99.30	97.61	**47.36**	50.13	33.58	14.02	12.99	31.62	55.76	30.27
DO 8bit	96.00	**98.31**	95.50	99.00	97.20	47.10	50.78	33.58	13.82	12.96	31.65	54.66	28.27
DO+	96.00	97.56	**97.50**	98.67	97.43	47.27	**51.89**	32.25	**15.39**	14.23	**32.81**	**56.45**	28.13
DO+ 8bit	96.00	**98.31**	**97.50**	99.33	97.79	46.93	51.66	34.47	13.71	**14.73**	32.30	55.61	28.27

Takeaway Messages

1. We propose **DualOptim+**, which introduces a **shared base state** to capture common representations and **decoupled delta states** to preserve task-specific residuals.
2. DualOptim+ is a **plug-and-play** framework applicable to any multi-objective optimization and optimizers with stored states.
3. Experiments confirm that DualOptim+ achieves a **superior trade-off between** forgetting efficacy and model utility in MU. Our method can also be extended to more general alignment tasks.

Codes on Github