Robust Binary Models by Pruning Randomly-Initialized Networks

Chen Liu* 1, Ziqi Zhao* 1, Sabine Süsstrunk1, Mathieu Salzmann1

¹ École Polytechnique Fédérale de Lausanne

March 22, 2022

Equal Contribution

C. Liu et. al. (EPFL)

. / 27



2 Motivation

3 Methodology



5 Conclusion



1 Introduction

Motivation

3 Methodology

4 Experiments

Conclusion

C. Liu et. al. (EPFL)

Definition (Robustness Problem)

Given a classification model $f(\theta, \mathbf{x}) : \Theta \times \mathbb{R}^M \to \mathbb{R}^K$ parameterized by θ , data points drawn from the distribution $(\mathbf{x}, y) \sim \mathcal{D}$ and loss function \mathcal{L} , robustness problem is formulation as follows:

$$\min_{\theta} \mathbb{E}_{(\mathbf{x}, y) \sim \mathcal{D}} \max_{\mathbf{x}' \in \mathcal{S}_{\epsilon}(\mathbf{x})} \mathcal{L}(f(\theta, \mathbf{x}'), y)$$
(1)

where $S_{\epsilon}(\mathbf{x})$ is called the adversarial budget: $S_{\epsilon}(\mathbf{x}) = {\mathbf{x}' || \mathbf{x} - \mathbf{x}' ||_{\infty} \le \epsilon}$.

Definition (Robustness Problem)

Given a classification model $f(\theta, \mathbf{x}) : \Theta \times \mathbb{R}^M \to \mathbb{R}^K$ parameterized by θ , data points drawn from the distribution $(\mathbf{x}, y) \sim \mathcal{D}$ and loss function \mathcal{L} , robustness problem is formulation as follows:

$$\min_{\theta} \mathbb{E}_{(\mathbf{x}, y) \sim \mathcal{D}} \max_{\mathbf{x}' \in \mathcal{S}_{\epsilon}(\mathbf{x})} \mathcal{L}(f(\theta, \mathbf{x}'), y)$$
(1)

where $S_{\epsilon}(\mathbf{x})$ is called the adversarial budget: $S_{\epsilon}(\mathbf{x}) = {\mathbf{x}' || \mathbf{x} - \mathbf{x}' ||_{\infty} \le \epsilon}$.

Adversarial Training (AT): generate optimal x' and then optimize θ on x'.

(4) E (4) E (4)

• For small model, adversarial training first fails to converge while vanilla training still succeed to obtain non-trivial performance.

• • = • • = •

- For small model, adversarial training first fails to converge while vanilla training still succeed to obtain non-trivial performance.
- For big model, performance of vanilla training first saturates with increasing model size while we still see improvement in adversarial training.

A B M A B M



Introduction

2 Motivation

3 Methodology

4 Experiments

Conclusion

C. Liu et. al. (EPFL)

- Related works
 - Adversarial training and model compression (pruning, quantization) at the same time. ^{1 2}
 - Pre-train, compression, fine-tune. ³

¹Gui, et. al. "Model compression with adversarial robustness: A unified optimization framework". NeurIPS 2019. ²Ye, et. al. "Adversarial robustness vs. model compression, or both". ICCV 2019. ³Solume at al. "Hudrey Druging adversarially robustness results accurate accurate adversarial robustness."

³Sehwa, et. al. "Hydra: Pruning adversarially robust neural networks". NeurIPS 2019. 🖬 २ २ 🖀 २ २ 🛓 🔊 ५

C. Liu et. al. (EPFL)

• Related works

- Adversarial training and model compression (pruning, quantization) at the same time. ^{1 2}
- Pre-train, compression, fine-tune. ³

Our proposed method: compression (pruning) itself as a way of training.

C. Liu et. al. (EPFL)

Related works

- Adversarial training and model compression (pruning, quantization) at the same time. ^{1 2}
- Pre-train, compression, fine-tune. ³

Our proposed method: **compression (pruning) itself as a way of training.** Adversarial training learns parameters, our method learns model architectures.

¹Gui, et. al. "Model compression with adversarial robustness: A unified optimization framework". NeurIPS 2019. ²Ye, et. al. "Adversarial robustness vs. model compression, or both". ICCV 2019. ³Sehwa, et. al. "Hydra: Pruning adversarially robust neural networks". NeurIPS 2019. □ > < ⑦ > < ≥ > < ≥ > ≥

C. Liu et. al. (EPFL)

Lottery Ticket Hypothesis⁴: Overparameterized neural networks contain sparse subnetworks that can be trained in isolation to achieve competitive performance. These subnetworks are called *winning tickets*.

⁴Frankle, et. al. "The lottery ticket hypothesis: Finding sparse, trainable neural networks". ICLR 2019.
⁵Ramanujan, et. al. "What's hidden in a randomly weighted neural network". CVPR 2020.

C. Liu et. al. (EPFL)

Robust Binary Models

Lottery Ticket Hypothesis⁴: Overparameterized neural networks contain sparse subnetworks that can be trained in isolation to achieve competitive performance. These subnetworks are called *winning tickets*.

Strong Lottery Ticket Hypothesis⁵**:** There exist winning tickets with competitive performance even without training.

⁴Frankle, et. al. "The lottery ticket hypothesis: Finding sparse, trainable neural networks". ICLR 2019.
⁵Ramanujan, et. al. "What's hidden in a randomly weighted neural network". CVPR 2020.

C. Liu et. al. (EPFL)

Robust Binary Models

Lottery Ticket Hypothesis⁴: Overparameterized neural networks contain sparse subnetworks that can be trained in isolation to achieve competitive performance. These subnetworks are called *winning tickets*.

Strong Lottery Ticket Hypothesis⁵**:** There exist winning tickets with competitive performance even without training.

Strong lottery ticket hypothesis in the context of adversarial training: Finding robust models in random-initialized networks.



Introduction

2 Motivation



4 Experiments

5 Conclusion

C. Liu et. al. (EPFL)

Definition (Adversarial Edge-Popup Problem)

Given a classification model $f(\theta, \mathbf{x}) : \Theta \times \mathbb{R}^M \to \mathbb{R}^K$ with initial parameter θ , data points drawn from the distribution $(\mathbf{x}, y) \sim \mathcal{D}$ and loss function \mathcal{L} , we aim to find the optimal subnetwork with mask $\mathbf{m} \in \{0, 1\}^{|\Theta|}$ to solve the following problem:

$$\min_{\mathbf{m} \in \{0,1\}^{|\Theta|}} \mathbb{E}_{(\mathbf{x},y) \sim \mathcal{D}} \max_{\mathbf{x}' \in \mathcal{S}_{\epsilon}(\mathbf{x})} \mathcal{L}(f(\theta \odot \mathbf{m}, \mathbf{x}'), y)$$

where $\mathcal{S}_{\epsilon}(\mathbf{x})$ is the adversarial budget and \odot the elementwise multiplication.

(2)

Definition (Adversarial Edge-Popup Problem)

Given a classification model $f(\theta, \mathbf{x}) : \Theta \times \mathbb{R}^M \to \mathbb{R}^K$ with initial parameter θ , data points drawn from the distribution $(\mathbf{x}, y) \sim \mathcal{D}$ and loss function \mathcal{L} , we aim to find the optimal subnetwork with mask $\mathbf{m} \in \{0, 1\}^{|\Theta|}$ to solve the following problem:

$$\min_{\mathbf{m} \in \{0,1\}^{|\Theta|}} \mathbb{E}_{(\mathbf{x},y) \sim \mathcal{D}} \max_{\mathbf{x}' \in \mathcal{S}_{\epsilon}(\mathbf{x})} \mathcal{L}(f(\theta \odot \mathbf{m}, \mathbf{x}'), y)$$

where $\mathcal{S}_{\epsilon}(\mathbf{x})$ is the adversarial budget and \odot the elementwise multiplication.

Instead of optimization parameter θ , we optimize the architecture represented by \mathbf{m} .

▲圖 ▶ ▲ 国 ▶ ▲ 国 ▶

(2)

Definition (Adversarial Edge-Popup Problem)

Given a classification model $f(\theta, \mathbf{x}) : \Theta \times \mathbb{R}^M \to \mathbb{R}^K$ with initial parameter θ , data points drawn from the distribution $(\mathbf{x}, y) \sim \mathcal{D}$ and loss function \mathcal{L} , we aim to find the optimal subnetwork with mask $\mathbf{m} \in \{0, 1\}^{|\Theta|}$ to solve the following problem:

$$\min_{\mathbf{m} \in \{0,1\}^{|\Theta|}} \mathbb{E}_{(\mathbf{x},y) \sim \mathcal{D}} \max_{\mathbf{x}' \in \mathcal{S}_{\epsilon}(\mathbf{x})} \mathcal{L}(f(\theta \odot \mathbf{m}, \mathbf{x}'), y)$$

where $\mathcal{S}_{\epsilon}(\mathbf{x})$ is the adversarial budget and \odot the elementwise multiplication.

Instead of optimization parameter θ , we optimize the architecture represented by **m**. Discrete optimization?

A (1) < A (2) < A (2) </p>

(2)

$$\min_{\mathbf{m} \in \{0,1\}^{|\Theta|}} \mathbb{E}_{(\mathbf{x},y) \sim \mathcal{D}} \max_{\mathbf{x}' \in \mathcal{S}_{\epsilon}(\mathbf{x})} \mathcal{L}(f(\theta \odot \mathbf{m}, \mathbf{x}'), y)$$

< □ ▷ < □ ▷ < □ ▷ < □ ▷ < □ ▷
March 22, 2022

$$\min_{\mathbf{m}\in\{0,1\}^{|\Theta|}} \mathbb{E}_{(\mathbf{x},y)\sim\mathcal{D}} \max_{\mathbf{x}'\in\mathcal{S}_{\epsilon}(\mathbf{x})} \mathcal{L}(f(\theta\odot\mathbf{m},\mathbf{x}'),y)$$

Trainable continuous alternative variable 'score' **s** to replace **m**: $\mathbf{m} = M(\mathbf{s})$

- Forward pass: elements with top k scores are assigned 1 in \mathbf{m} and 0 otherwise.
 - The value of k depends on the pruning policy.
- Backward pass: treated as the identity function ('gradient goes through').

$$\min_{\mathbf{m}\in\{0,1\}^{|\Theta|}} \mathbb{E}_{(\mathbf{x},y)\sim\mathcal{D}} \max_{\mathbf{x}'\in\mathcal{S}_{\epsilon}(\mathbf{x})} \mathcal{L}(f(\theta\odot\mathbf{m},\mathbf{x}'),y)$$

Trainable continuous alternative variable 'score' **s** to replace **m**: $\mathbf{m} = M(\mathbf{s})$

- Forward pass: elements with top k scores are assigned 1 in \mathbf{m} and 0 otherwise.
 - The value of k depends on the pruning policy.
- Backward pass: treated as the identity function ('gradient goes through').
- Approximation in the backward does not affect adversarial examples generation.
- Can be combined with any variant of adversarial training.
 - Accelerated version, adversarial training with regularization.

► < ∃ ►</p>

Algorithm 1: Edge pop-up algorithm for adversarial robustness.

Input: training set \mathcal{D} , batch size B, PGD step size α and iteration number n, adversarial budget \mathcal{S}_{ϵ} , pruning rate r, mask function M, the optimizer. Random initialize the model parameters θ and the scores \mathbf{s} . for Sample a mini-batch $\{\mathbf{x}_i, y_i\}_{i=1}^B \sim \mathcal{D}$ do for i = 1, 2, ..., B do Sample a random noise δ within the adversarial budget \mathcal{S}_{ϵ} . $\mathbf{x}_i^{(0)} = \mathbf{x}_i + \delta$ for $\mathbf{j} = 1, 2, ..., \mathbf{n}$ do $\mathbf{x}_i^{(j)} = \mathbf{x}_i^{(j-1)} + \alpha \nabla_{\mathbf{x}_i^{(j-1)}} \mathcal{L}(f(\theta \odot M(\mathbf{s}, r), \mathbf{x}_i^{(j-1)}), y_i)$ $\mathbf{x}_i^{(j)} = \mathbf{x}_i + \Pi_{\mathcal{S}_{\epsilon}} \left(\mathbf{x}_i^{(j)} - \mathbf{x}_i\right)$

end for

end for

Calculate the gradient $\mathbf{g} = \frac{1}{B} \sum_{i=1}^{B} \nabla_{\mathbf{s}} \mathcal{L}(f(\theta \odot M(\mathbf{s}, r), \mathbf{x}_{i}^{(n)}), y_{i})$ Update the score \mathbf{s} using the optimizer.

end for

Output: the pruning mask $M(\mathbf{s}, r)$.

• • = • • = •

Two pruning strategies for pruning rate r in existing works:

- Global pruning: top $(1 r)|\Theta|$ elements with the largest scores.
- Fixed pruning rate: prune parameters of each layer with the same proportion.

Two pruning strategies for pruning rate r in existing works:

- Global pruning: top $(1 r)|\Theta|$ elements with the largest scores.
- Fixed pruning rate: prune parameters of each layer with the same proportion.

However... these do not achieve ideal performance in the context of adversarial training.

▶ < ∃ ▶</p>

< □ > < 向

• • = • • = •

Fixed pruning rate

$$\frac{m_1}{n_1} = \frac{m_2}{n_2} = \dots = \frac{m_L}{n_L}$$

Fixed pruning rate

$$\frac{m_1}{n_1} = \frac{m_2}{n_2} = \dots = \frac{m_L}{n_L}$$

Maximal of $\prod_{i=1}^L \binom{n_i}{m_i}$

C. Liu et. al. (EPFL)

Fixed pruning rate

$$\frac{m_1}{n_1} = \frac{m_2}{n_2} = \dots = \frac{m_L}{n_L}$$

Maximal of $\prod_{i=1}^L \binom{n_i}{m_i}$
Maximize the search space for **m**.

(4) E (4) E (4)

Fixed pruning rate

$$\frac{m_1}{n_1} = \frac{m_2}{n_2} = \dots = \frac{m_L}{n_L}$$

Maximal of $\prod_{i=1}^L \binom{n_i}{m_i}$
Maximize the search space for **m**.

Maximize # input-output paths.

Fixed pruning rate

$$\frac{m_1}{n_1} = \frac{m_2}{n_2} = \dots = \frac{m_L}{n_L}$$

Maximal of $\prod_{i=1}^L \binom{n_i}{m_i}$
Maximize the search space for **m**.

Maximal of $\prod_{i=1}^{L} m_i$

Maximize # input-output paths.

Fixed pruning rate

Fixed parameter number

$$\frac{m_1}{n_1} = \frac{m_2}{n_2} = \dots = \frac{m_L}{n_L}$$

Maximal of $\prod_{i=1}^L \binom{n_i}{m_i}$
Maximize the search space for **m**.

 $m_1 = m_2 = \dots = m_L$

Maximal of $\prod_{i=1}^{L} m_i$

Maximize # input-output paths.

A B > A B >

э.

Fixed pruning rate

Fixed parameter number

$$\frac{m_1}{n_1} = \frac{m_2}{n_2} = \dots = \frac{m_L}{n_L}$$

Maximal of $\prod_{i=1}^L \binom{n_i}{m_i}$
Maximize the search space for **m**.
Poor performance when *r* **is big.**

 $m_1 = m_2 = \ldots = m_l$

Maximal of $\prod_{i=1}^{L} m_i$

Maximize # input-output paths. Poor performance when r is small.

m.

A B M A B M

3

Adaptive pruning strategy: $\frac{m_1}{n_1^p} = \frac{m_2}{n_2^p} = ... = \frac{m_L}{n_L^p}$

- Degrade to fixed pruning rate and fixed parameter number with p = 1 and p = 0.
- Larger $r \rightarrow$ smaller p; smaller $r \rightarrow$ larger p.

A B < A B </p>

3



C.	Liu	et.	al.	(EP	FL

16 / 27

• The scale of initializaiton is voided by normalization layers.

- The scale of initializaiton is voided by normalization layers.
- Each linear layer should be followed by a normalization layer, including the last layer.

(4) E (4) E (4)

- The scale of initializaiton is voided by normalization layers.
- Each linear layer should be followed by a normalization layer, including the last layer.

Two advantages of models including the last normalization layer.

▶ < ∃ ▶</p>

- The scale of initializaiton is voided by normalization layers.
- Each linear layer should be followed by a normalization layer, including the last layer.

Two advantages of models including the last normalization layer.

- Arbitrary initialization scale of random parameters.
 - Binary initialization: save 1/2 and 1/4 computational time in forward and backward.

▶ < ∃ ▶</p>

- The scale of initializaiton is voided by normalization layers.
- Each linear layer should be followed by a normalization layer, including the last layer.

Two advantages of models including the last normalization layer.

- Arbitrary initialization scale of random parameters.
 - Binary initialization: save 1/2 and 1/4 computational time in forward and backward.
- Insensitive to the initialization scale of **m**.

→



Introduction

2 Motivation

3 Methodology



5 Conclusion

C. Liu et. al. (EPFL)

Prune	<i>r</i> = 0.5	<i>r</i> = 0.8	<i>r</i> = 0.9	<i>r</i> = 0.95	<i>r</i> = 0.99	<i>r</i> = 0.995	<i>r</i> = 0.998
Strategy							
p = 0.0	2.16	6.86	23.01	41.61	44.60	40.70	34.97
ho=0.1	4.35	15.03	28.12	42.65	44.88	40.97	33.09
p = 0.2	8.01	19.21	27.99	43.72	42.92	40.52	32.99
p=0.5	9.21	32.70	42.84	43.62	42.45	40.55	30.08
p = 0.8	28.90	41.51	43.64	43.88	39.12	33.61	28.07
ho=0.9	39.09	41.71	43.07	42.28	38.68	33.89	17.43
ho=1.0	42.85	43.23	42.13	41.12	34.57	26.67	20.56

Pruning rate r and p in adaptive pruning strategy.

Table: Robust accuracy (in %) on the CIFAR10 test set under different pruning rates r and values of p in *adaptive pruning*. The best result for each pruning rate is marked in bold.

Advantages with the last batch normalization layer. Assume the score **s** is initialized based on $\mathcal{N}(0, a^2)$.

Madal	Valu	e of <i>a</i> in sc	ore initializ	ation
Model	0.001	0.01	0.1	1
no LBN	33.08	39.96	41.01	31.04
LBN	45.06	44.88	44.63	44.41

Table: Robust accuracy (in %) on the CIFAR10 test set for models with and without the last batch normalization layer (LBN) under different values of a for score s initialization. The best results are marked in bold.

3

Experiments Ablation Study

Advantages with the last batch normalization layer.

Prune	Signe	Signed KC		ary
Strategy	no LBN	LBN	no LBN	LBN
p = 0.0	39.38	42.83	40.94	44.65
ho=0.1	39.62	45.01	41.01	45.06
p = 0.2	36.66	45.04	37.85	41.58
ho=0.5	39.98	42.64	40.61	39.95
ho=0.8	37.96	41.71	35.15	38.95
ho=0.9	34.75	40.14	35.64	35.81
p = 1.0	36.88	39.32	30.02	30.62

Table: Robust accuracy (in %) on the CIFAR10 test set with the Signed Kaiming Constant (Signed KC) and the binary initialization. We include models both with and without the last batch normalization layer (LBN). The best results are marked in bold.

C. E.u. Ct. u (E	C.	Liu	et.	al.	(EPFL
------------------	----	-----	-----	-----	-------

Experiment

Comparison with Existing Works

Mathad	A web it a sturre	Pruning	Pruning	CIFA	AR10 CIFAR100		R100
Method	Architecture	Strategy	Rate	FP	Binary	FP	Binary
AT	ResNet34	-	-	43.26	40.34	36.63	26.49
AT	ResNet34-LBN	-	-	42.39	39.58	35.15	32.98
- HYDRA	ResNet34	p = 0.1	0.99	42.73	29.28	33.00	23.60
HYDRA	ResNet34	p = 1.0	0.99	40.51	26.40	31.09	18.24
HYDRA	ResNet34-LBN	p = 0.1	0.99	40.55	33.99	13.63	24.69
HYDRA	ResNet34-LBN	p = 1.0	0.99	32.93	26.23	29.96	17.75
ATMC	ResNet34	Global	0.99	34.14	25.62	25.10	11.09
ATMC	ResNet34	ho=0.1	0.99	34.58	24.65	25.37	11.04
ATMC	ResNet34	p = 1.0	0.99	30.50	20.21	22.28	2.53
ATMC	ResNet34-LBN	Global	0.99	33.55	19.01	23.16	15.73
ATMC	ResNet34-LBN	ho=0.1	0.99	31.61	22.88	25.16	17.33
ATMC	ResNet34-LBN	p = 1.0	0.99	27.88	13.22	22.12	9.55
Ours	ResNet34-LBN	p = 0.1	0.99	-	45.06	-	34.83
Ours	ResNet34-LBN	ho=1.0	0.99	-	34.57	-	26.32
Ours (Fast)	ResNet34-LBN	ho=0.1	0.99	-	40.77	-	34.45
Ours (Fast)	ResNet34-LBN	p = 1.0	0.99	-	29.68	-	24.97

Table: Robust accuracy (in %) on the CIFAR10 and CIFAR100 for AT, HYDRA, ATMC and our proposed method. The best results for full precision (FP) models are underlined; the best results for binary models are marked in bold.

C. Liu et. al. (EPFL)

21/27

Experiment

Comparison with Existing Works

Mathad	A web it a sturre	Pruning	Pruning	CIFA	AR10	CIFAR100	
Method	Architecture	Strategy	Rate	FP	Binary	FP	Binary
AT	ResNet34	-	-	80.99	74.37	<u>61.48</u>	47.87
AT	ResNet34-LBN	-	-	80.96	74.17	57.73	60.08
- HYDRA	ResNet34	p = 0.1	0.99	75.31	62.09	55.92	45.96
HYDRA	ResNet34	p = 1.0	0.99	73.89	59.69	54.88	37.84
HYDRA	ResNet34-LBN	p = 0.1	0.99	74.37	68.43	53.65	46.09
HYDRA	ResNet34-LBN	p = 1.0	0.99	66.59	57.15	26.64	37.16
ATMC	ResNet34	Global	0.99	81.85	72.97	57.15	36.39
ATMC	ResNet34	p = 0.1	0.99	81.37	73.34	59.99	32.68
ATMC	ResNet34	p = 1.0	0.99	74.33	57.01	54.06	5.19
ATMC	ResNet34-LBN	Global	0.99	80.72	68.25	55.69	40.51
ATMC	ResNet34-LBN	ho=0.1	0.99	76.39	51.31	57.34	43.97
ATMC	ResNet34-LBN	ho = 1.0	0.99	69.83	19.26	49.69	28.78
Ours	ResNet34-LBN	p = 0.1	0.99	-	76.59	-	60.16
Ours	ResNet34-LBN	p = 1.0	0.99	-	65.70	-	47.21
Ours (Fast)	ResNet34-LBN	ho=0.1	0.99	-	81.63	-	63.73
Ours (Fast)	ResNet34-LBN	p=1.0	0.99	-	70.72	-	50.98

Table: Clean accuracy (in %) on the CIFAR10 and CIFAR100 for AT, HYDRA, ATMC and our proposed method. The best results for full precision (FP) models are underlined; the best results for binary models are marked in bold.

C. Liu et. al. (EPFL)

22 / 27

Experiments Structures of Pruned Subset Networks



Figure: Mask visualization of the weight of a random convolutional layer in our model. The parameters retained is highlighted as blue dots. The dimension of the convolutional kernel is (r_{out} , r_{in} , 3, 3). We reshape this kernel in rectangle of shape ($r_{out} \times 3$, $r_{in} \times 3$). Channels with no remaining weight are colored orange. The top bar indicates whether the channel is empty (white) or not (blue).

C. Liu et. al. (EPFL)

23 / 27

3

Experiments Structures of Pruned Subset Networks



Figure: Distribution of weights in two consecutive layers. In layer1 (left), the masks are reshaped into ($r_{out} \times 3$, $r_{in} \times 3$) while masks in layer2 (right) are reshaped into ($r_{in} \times 3$, $r_{out} \times 3$). The output channels totally pruned in layer1 and the input channels totally pruned in layer2 are highlighted as the white bars in the middle.

・ロト ・ 母 ト ・ ヨ ト ・ ヨ ト ・ ヨ



Introduction

2 Motivation

3 Methodology

4 Experiments

5 Conclusion

C. Liu et. al. (EPFL)

- We extend 'Strong Lottery Hypothesis' to robust learning.
- We propose 'adaptive pruning strategy' to improve the performance.
- We adapt the algorithm to binary networks to accelerate computation.

Unsolved challenges and potential future works.

- Automatic pruning strategy.
- Structured pruning on randomly-initialized network.

Thank You!

C. Liu et. al. (EPFL)

≺ □ ► < ⊡ ► < ≧ ► < ≧ ► March 22, 2022

27 / 27

3